A new bivariate control chart for monitoring the mean vector and covariance matrix simultaneously

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Abstract

A new bivariate single control chart is proposed to simultaneously monitor the process mean vector and the process covariance matrix. This chart, called CSDW chart, is based on the cumulative sum of the diagonal elements of Wishart distributed matrix. Through our average run length (ARL) comparison, the results of the simulation show that the new chart outperforms the joint T-square and |S| charts, the Max-MEWMA chart, and the VMAX. Examples are also given to illustrate the new chart procedure.

Keywords: control chart, the joint T-square and |S| charts, Max-MEWMA, VMAX, average run length

1 Introduction

Statistical process control (SPC) is one of the most effective approaches for quality improvement. A control chart is one of the most important tools often used to observe whether a process is in control or not. There are many situations where it is necessary to monitor a process with more than one quality characteristics simultaneously. Many multivariate control charts have been presented in the literature of quality control since Hotelling [1], being the first one, provided the T^2 statistic for monitoring the mean vector $\boldsymbol{\mu}$ of the process. The first multivariate control procedure for monitoring the covariance matrix $\boldsymbol{\Sigma}$ was derived from the generalized likelihood ratio test. For the bivariate process, Alt [2] developed the generalized variance statistic $|\mathbf{S}|$ to control the covariance matrix $\boldsymbol{\Sigma}$. Jackson and Wiley [3] mentioned that any multivariate control charts should possess four important properties, namely, they should answer these questions: (1) whether the process is in-control, (2) whether the specified probability of Type-Ⅰ error has been maintained, (3) whether the relationships between the variables have been taken into account, and (4) what the problem is if the process is out of control.

The substantial amount of researches in non-sequential multivariate quality control charts can be classified into four broad categories, namely, (1) Multivariate Shewhart Charts, (2) Multivariate Cumulative Sum (MCUSUM) charts, (3) Multivariate Exponentially Weighted Moving Average (MEWMA) Charts, and (4) Multivariate charts based on Artificial Neural Networks (ANN) [4].

Early research on multivariate Shewhart charts goes back to Hotelling's T^2 statistic [1]. It is the optimal statistic for detecting a general shift in the process mean vector for an individual multivariate observation [5]. The MCUSUM type control chart proposed by Crosier [6] and Pignatiello and Runger [7] is one of the four categories of multivariate charts mentioned in the above paragraph. Crosier [6] proposed the design procedures and average run lengths for two multivariate cumulative sum (MCUSUM) quality-control procedures. The first MCUSUM procedure reduces each multivariate observation to a scalar and then forms a CUSUM of the scalars. The second CUSUM procedure formed a CUSUM vector directly from the observations. These two procedures are compared with each other and with the multivariate Shewhart chart. The robustness of the procedures and combined Shewhart-CUSUM schemes are discussed. Pignatiello and Runger [7] considered several distinct approaches for controlling the mean of a multivariate normal process. They compared the performances of these approaches through estimating the average run length and presented the average run length results.

The performances of the MEWMA control charts are quite similar to those of the MCUSUM. In this category, several researchers proposed different procedures, to name a few see Lowry et al. [8], Prabhu and Runger [9], Yumin [10], Alessandro [11], Yeh et al. [12], and Fallah Nezhad [13]. Lowry et al. [8] proposed an extension of the exponentially weighted moving average (EWMA) control charts to the multivariate case. They studied the ARL performance of the MEWMA chart and compared it with the MCUSUM chart. They stated that their control charts was similar to the MCUSUM control chart in detecting a shift in the mean vector of a multivariate normal distribution. Yumin [10] proposed a new MEWMA control chart based on the principal components of the

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original variables. Alessandro [11] presented a one-side MEWMA control chart based on the restricted maximum likelihood estimator. Yeh et al. [12] introduced a new multivariate exponentially weighted moving average control chart designed to detect small changes in the variability of correlated multivariate quality characteristics. Fallah Nezhad [13] proposed a new approach to detect shifts of a multivariate quality control procedure. He compared the chart with a MCUSUM control chart and a MEWMA control chart based on the in- and out-of-control ARL. He concluded that the chart performs better than the other two methods in detecting shifts in the standard deviation and correlation.

Alt [2] reviewed multivariate quality control and concluded that an important area worthy of further research is to propose a single control chart for monitoring the process location and process dispersion at the same time. Some researchers have contributed to the theory and practical use of multivariate single control charts. Khoo [14] developed a control chart with the T^2 and $|\mathbf{S}|$ statistics to monitor both the bivariate process mean and variance. The performance of the new chart by means of its average run length (ARL) profiles was obtained by simulation studies. The proposed chart is insensitive to small changes in the multivariate process mean and variance. Chen et al. [15] proposed a single EWMA chart, named Max-MEWMA chart, to control the mean vector and the covariance matrix simultaneously. Their chart is more efficient than the joint T^2 and $|\mathbf{S}|$ charts in signalling small changes in the process. Machado and Costa [16] presented the use of two charts jointly, based on the non-central chi-square statistic for monitoring the mean vector and the covariance matrix of a bivariate process. The scheme is faster than the joint use of the T^2 and $|\mathbf{S}|$ charts in signalling small changes in a bivariate process mean. Costa and Machado [17] proposed the VMAX statistic to control the covariance matrix of multivariate process. The VMAX chart is always more efficient than the chart based on the generalized variance $|\mathbf{S}|$. Bersimis et al. [18] discussed the basic procedure for the implementation of multivariate statistical process control via control charting. They reviewed multivariate extensions for all kinds of univariate control charts, such as multivariate Shewhart-type control charts, multivariate CUSUM control charts and multivariate EWMA control charts. In addition, they reviewed unique procedure for the construction of multivariate control charts, based on multivariate statistical techniques, such as principal components analysis (PCA) and partial least squares (PLS). S. Y. Teh, Michael B. C. Khoo et al. [19] proposed a new GWMA chart, called the Max-GWMA chart, which uses a single statistic for a simultaneous monitoring of the process mean and variance. The statistic of the Max-GWMA chart is based on the maximum of the absolute values of two GWMA statistics, one for controlling the mean while the other the variance. They show that the Max-GWMA chart outperforms the combined GWMA chart, in terms of the average run length (ARL), standard deviation of the run length (SDRL) and diagnostic abilities performances. Xiaobei Shen, Fugee Tsung et al. [20] developed a new multivariate exponentially weighted moving average control chart for the monitor of the covariance matrices by integrating the classical L_2-norm-based test with a maximum-norm-based test. Numerical studies show that the new control chart affords more balanced performance under various shift directions than the existing ones and is thus an effective tool for multivariate SPC applications. Li-ping Liu, Jian-lan Zhong et al. [21] proposed a multivariate synthetic control chart for monitoring the changes in the covariance matrix of a multivariate process under multivariate normal distribution. The proposed control chart is a combination of the traditional control chart based on conditional entropy and the conforming run length chart. The other researchers, to name a few, see Bersimis et al. [22], Yeh et al. [23], Topalidou and Psarakis [24], and Butte and Tang [25].

In this study, the statistic based on the Wishart distribution is used to develop a bivariate single control chart. This new chart can be used to monitor the process mean vector and covariance matrix simultaneously. The average run length (ARL) performance of the new chart is studied and we find that when compared with the joint T^2 and $|\mathbf{S}|$ charts, the Max-MEWMA chart, and the VMAX, the new chart is faster in detecting changes in the process. An example is given to illustrate the implementation of the new chart.

The rest of the paper is organized as follows. In section 2 we present the new control chart. The performance of the CSDW chart is discussed in Section 3. Section 4 provides comparisons between four control charts. An example is presented to illustrate the implementation and the application of the proposed procedure in Section 5. Finally, the last section summarizes the paper.

2 The new control chart

The mean vector $\boldsymbol{\mu_0}$ and covariance matrix $\boldsymbol{\Sigma}_0$ of a multivariate process can be estimated from a large number of preliminary samples of the product and can therefore be assumed to be known [26,27]. Assuming that $\mathbf{X}_1, \mathbf{X}_2$, …, $\mathbf{X}_j$, $j = 1, 2, ..., p$, is size n samples of p quality characteristics process, where $\mathbf{X}_j \sim N_p(\boldsymbol{\mu}_0, \boldsymbol{\Sigma}_0)$. We suppose that the random $\mathbf{X}_j$ is independent of each other, both within the sample and between the samples. Here we need to standardize vector $\mathbf{X}_j$ in order to meet the objective of the article. Then the random matrix

$\mathbf{W}=\sum_{i=1}^{n}\mathbf{X}_j\mathbf{X}_j^{'}$ follows a Wishart distribution with n degrees of freedom, that is, $\mathbf{W}\sim\mathbf{W}_n(\mathbf{\Sigma}_0)$

Its probability density function is

$$f_m(\mathbf{W})=\frac{|\mathbf{W}|^{(n-p-1)/2}\exp\left\{-\frac{1}{2}tr(\mathbf{W}\mathbf{\Sigma}_0^{-1})\right\}}{2^{\frac{np}{2}}\pi^{\frac{p(p-1)}{4}}|\mathbf{\Sigma}|^{\frac{n}{2}}\prod_{i=1}^{p}\Gamma(\frac{1}{2}(n-i+1))},$$

where $\Gamma(\cdot)$ and $tr(\cdot)$ are the gamma function and the trace of the matrix, respectively.

We consider monitoring the process mean vector and covariance matrix simultaneously. We define our statistic for monitoring the process mean vector and covariance matrix as $T_{CSDW}=tr(\mathbf{W})$, which is the sum of the diagonal elements, that is, the square cumulative sum of each subgroup sample data, in the matrix $\mathbf{W}$.

We call the new chart based on T_{CSDW} the CSDW chart. Because T_{CSDW} is positive, our CSDW chart needs an upper control limit (UCL) only, which is obtained through simulation.

Here we suppose that increasing changes in the variability and changes in the mean vector are considered. Another assumption is that the correlations between characteristics are unchanged.

3 The performance of the CSDW chart

A simulation is considered to study the ARL performance of the new chart. The study is based on the chart with a subgroup size of $n=5$ and an ARL_0 value of 200. The UCL values for the chart are obtained through simulation. Here, we assume that the in-control mean vector is $\mathbf{\mu}_0=(0,0)^{'}$ while the in-control covariance matrix is $\mathbf{\Sigma}_0=\begin{bmatrix}1 & \rho\\ \rho & 1\end{bmatrix}$. For the in-control situation, the mean vector shifts from $\mathbf{\mu}_0$ to $\mathbf{\mu}_S$ and the covariance matrix shifts from $\mathbf{\Sigma}_0$ to $\mathbf{\Sigma}_S$. According to the assuming, we assume, without loss of generality [15], that the mean vector shifts from $\mathbf{\mu}_0=(0,0)^{'}$ to $\mathbf{\mu}_s=(0,b)^{'}$ and the covariance matrix shifts from $\mathbf{\Sigma}_0=\begin{bmatrix}1 & \rho\\ \rho & 1\end{bmatrix}$ to $\Sigma_s=a^2\begin{bmatrix}1 & \rho\\ \rho & 1\end{bmatrix}, -1<\rho<1$, and $a>1$.

Here, we assume that the correlation between the two quality characteristics is still equal to ρ after the covariance matrix has been changed.

TABLE 1 ARL values of the CSDW chart when $p=2$ and $n=5$ in the simulation. The in-control ARL is 200

ρ		b						
		0.00	0.50	1.00	1.50	2.00	2.50	3.00
a=1.00	0.0^1	201.6	78.1	14.5	3.5	1.5	1.1	1.0
	0.3^2	200.1	86.9	17.6	4.2	1.6	1.1	1.0
	0.6^3	199.5	103.3	25.0	5.9	2.0	1.2	1.0
	0.9^4	201.5	115.5	33.9	8.9	2.8	1.3	1.0
a=1.50	0.0	2.9	2.6	1.9	1.4	1.2	1.0	1.0
	0.3	3.3	2.9	2.1	1.5	1.2	1.1	1.0
	0.6	4.1	3.6	2.6	1.8	1.3	1.1	1.0
	0.9	5.0	4.4	3.2	2.2	1.5	1.2	1.0
a=2.00	0.0	1.3	1.2	1.2	1.1	1.1	1.0	1.0
	0.3	1.3	1.3	1.2	1.1	1.1	1.0	1.0
	0.6	1.5	1.5	1.4	1.2	1.1	1.0	1.0
	0.9	1.8	1.7	1.6	1.4	1.2	1.1	1.0
a=2.50	0.0	1.1	1.1	1.0	1.0	1.0	1.0	1.0
	0.3	1.1	1.1	1.1	1.0	1.0	1.0	1.0
	0.6	1.1	1.1	1.1	1.1	1.0	1.0	1.0
	0.9	1.3	1.2	1.2	1.1	1.1	1.0	1.0
a=3.00	0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.3	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.6	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	0.9	1.1	1.1	1.1	1.1	1.0	1.0	1.0

UCL=25.18[1], UCL=26.35[2], UCL=29.04[3], UCL=32.35[4]

The ARL performances for the combinations are shown in Table 1.

We see from Table 1 that as a whole the ARL value reduces as the magnitude of the shift in the mean vector increases and/or the variability in the covariance increases. For example, from Table 1, the ARL profile for, $a=1.50$, $\rho=0.3$, and $b=0.00$, 0.50, 1.00, 1.50, 2.00, 2.50 and 3.00 are 3.3, 2.9, 2.1 1.5, 1.2, 1.1 and 1.0. In this situation, the ARL values display a downward trend as the mean shift b increases. The ARL values increase slightly as the correlation ρ increases, e.g. from Table 1, the ARL profile, for, $a = 2.00$, $b = 1.00$, and $\rho = 0.0$, 0.3, 0.6 and 0.9 are 1.2, 1.2, 1.4 and 1.6, where the ARL values display a slight upward trend as ρ increases.

4 The performance of the proposed control chart

Here, the goal is to design a single control chart to monitor the process mean vector and the covariance matrix of a bivariate process simultaneously. Let $\boldsymbol{\mu}_0$, $\boldsymbol{\mu}_S$ and $\boldsymbol{\Sigma}_0$ be the same as those defined in Section 3 and the case considered here be the same as the case in Section 3.

In this section, some simulation studies are performed to evaluate the performance of the proposed procedure, based on the in-control average run length (ARL_0) criterion [28]. Then the studies compare the proposed chart's out-of-control average run length (ARL_1) [28] with those from the joint T^2 and $|\mathbf{S}|$ charts [14], the Max-MEWMA chart [15], and the VMAX_chart [17], for monitoring the mean vector and/or covariance matrix simultaneously.

For the comparison studies with an intended ARL_0 of 200, the out-of-control ARLs of the proposed chart, as well as the other procedures are estimated by 20,000 independent replications, in each of the different scenarios of process changes. In what follows, the performances of these procedures are compared, for the different changes in the process mean vector and covariance matrix.

The ARL_1 value of the presented procedure, as well as the joint T^2 and $|\mathbf{S}|$ charts [14], the Max-MEWMA chart [15], and the VMAX statistic [17] are estimated. Random data are generated from a bivariate standard normal distribution. Let the quality characteristics be random variables X_1 and X_2, where various coefficient of correlation are considered. The third up to the seventeenth columns in Table 2 show the ARL_1 values for the methods under consideration. The third up to the sixth column of Table 3 show ARL_1 value of the VMAX chart when just monitoring the covariance matrix.

We compare the ARL's of the CSDW chart with the ARL's of the two charts from Chen et al. [15], and the ARL's of the VMAX chart from Costa and Machado [17]. The results of Table 2 show the proposed method does outperforms the joint T^2 and $|\mathbf{S}|$ charts [14], and the Max-MEWMA [15] methods. The results of Table 3 show that the proposed method does a bit better than the VMAX chart [17] based on the standardized sample variance of quality characteristics.

5 Implementation and examples

We can implement a CSDW chart according to the steps summarized as follows:

1) If process parameters, $\boldsymbol{\mu}_0$ and $\boldsymbol{\Sigma}_0$, are unknown, substitute $\overline{\overline{\mathbf{X}}}$ for $\boldsymbol{\mu}_0$ and $\overline{\mathbf{S}}$ for $\boldsymbol{\Sigma}_0$, where $\overline{\overline{\mathbf{X}}}$ and $\overline{\mathbf{S}}$ are the grand mean vector and the average sample covariance matrix, respectively. They are estimated from a reliable historical data set taken from a stable process.

2) Standardize the data samples.

3) Compute the matrix $\mathbf{W}$ and the trace T_{CSDW} of the matrix $\mathbf{W}$, for each subgroup.

4) Find the UCL using simulation based on a desired ARL_0.

5) If $T_{CSDW} \leq$ UCL, plot a point on the chart at that time.

6) If T_{CSDW} >UCL, an out-of-control process is signalled. One needs to investigate the cause(s) associated with each out-of-control point. These out-of-control points must be removed as soon as possible in order to bring the process back into an in-control state.

Example: In this subsection, an example partially derived from Crosier [6] is provided to explain further the CSDW chart developed above. In Crosier [6], a bivariate normal distribution is considered with unit variance and a correlation coefficient of 0.5. The in-control process mean vector and covariance matrix are given by $\boldsymbol{\mu}_0 = (0,0)'$ and $\boldsymbol{\Sigma}_0 = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}$, respectively.

For our purpose, in Table 4, we initially generate 20 samples of size $n = 5$, in which the first 10 samples were simulated with the condition that the process is in control and the remaining 10 samples were simulated considering that the assignable cause has changed the mean vector from $\boldsymbol{\mu}_0$ to $\boldsymbol{\mu}_S$ and the covariance matrix from $\boldsymbol{\Sigma}_0$ to $\boldsymbol{\Sigma}_s = 1.5 \times \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}$, that is, a=1.5 and b=0.5. Then we can obtain T_{CSDW}.

TABLE 2 ARL's of the CSDW chart, the Max-MEWMA chart and the combination of T^2 chart and $|\mathbf{S}|$ chart when $p=2$ and $n=5$ in the Case of Section 3

ρ		b														
		Proposed method					Max-MEWMA chart[1]					Combination chart[1]				
		0.00	0.50	1.00	2.00	3.00	0.00	0.50	1.00	2.00	3.00	0.00	0.50	1.00	2.00	3.00
a=1.00	0.0	201.6	78.1	14.5	1.5	1.0	199.4	11.5	3.9	2.0	1.2	200.2	48.4	6.6	1.1	1.0
	0.3	200.1	86.9	17.6	1.6	1.0	199.5	11.5	3.7	1.9	1.1	200.2	43.6	5.7	1.1	1.0
	0.6	199.5	103.3	25.0	2.0	1.0	199.5	7.7	3.0	1.6	1.0	200.2	28.2	3.2	1.0	1.0
	0.9	201.5	115.5	33.9	2.8	1.0	199.5	3.3	1.8	1.0	1.0	200.2	4.2	1.0	1.0	1.0
a=1.50	0.0	2.9	2.6	1.9	1.2	1.0	4.2	3.8	3.1	1.9	1.3	5.1	4.1	2.6	1.2	1.2
	0.3	3.3	2.9	2.1	1.2	1.0	4.2	3.8	3.0	1.8	1.2	5.1	4.0	2.4	1.2	1.2
	0.6	4.1	3.6	2.6	1.3	1.0	4.2	3.7	2.7	1.6	1.0	5.1	3.7	2.0	1.1	1.1
	0.9	5.0	4.4	3.2	1.5	1.0	4.2	2.8	1.7	1.0	1.0	5.1	2.2	1.1	1.0	1.0
a=2.00	0.0	1.3	1.2	1.2	1.1	1.0	2.3	2.3	2.2	1.8	1.3	1.7	1.6	1.4	1.1	1.0
	0.3	1.3	1.3	1.2	1.1	1.0	2.3	2.3	2.1	1.7	1.3	1.7	1.6	1.4	1.1	1.0
	0.6	1.5	1.5	1.4	1.1	1.0	2.3	2.3	2.1	1.5	1.1	1.7	1.6	1.3	1.1	1.0
	0.9	1.8	1.7	1.6	1.2	1.0	2.3	2.1	1.6	1.0	1.0	1.7	1.4	1.1	1.0	1.0
a=2.50	0.0	1.1	1.1	1.0	1.0	1.0	2.0	2.0	1.9	1.7	1.3	1.2	1.2	1.1	1.1	1.0
	0.3	1.1	1.1	1.1	1.0	1.0	2.0	2.0	1.9	1.6	1.3	1.2	1.2	1.1	1.1	1.0
	0.6	1.1	1.1	1.1	1.0	1.0	2.0	2.0	1.9	1.5	1.1	1.2	1.2	1.1	1.0	1.0
	0.9	1.3	1.2	1.2	1.1	1.0	2.0	1.9	1.5	1.0	1.0	1.2	1.1	1.0	1.0	1.0
a=3.00	0.0	1.0	1.0	1.0	1.0	1.0	1.9	1.9	1.8	1.6	1.3	1.1	1.1	1.1	1.0	1.0
	0.3	1.0	1.0	1.0	1.0	1.0	1.9	1.9	1.8	1.6	1.3	1.1	1.1	1.1	1.0	1.0
	0.6	1.0	1.0	1.0	1.0	1.0	1.9	1.8	1.7	1.4	1.2	1.1	1.1	1.0	1.0	1.0
	0.9	1.1	1.1	1.1	1.0	1.0	1.9	1.8	1.5	1.1	1.1	1.1	1.0	1.0	1.0	1.0

[1]The data from Chen et al.[15]

TABLE 3 ARL's of the CSDW chart, and the VMAX chart when $p=2$ and $n=5$ in the Case of Section 3

		ρ			
		0.0	0.3	0.5	0.7
a^2	UCL_{CSDW}	25.18	26.35	28.05	30.15
	UCL_{VMAX}	3.677	3.675	3.668	3.646
1.0	VMAX	200.0	200.0	200.0	200.0
	CSDW	200.0	200.0	200.0	200.0
1.1		139.1	139.3	139.7	140.5
		132.8	136.1	138.0	139.98
1.2		101.6	101.8	102.4	103.6
		92.1	97.5	102.9	103.7
1.3		77.1	77.4	78.0	79.3
		67.7	72.7	78.0	80.3
1.4		60.5	60.8	61.4	62.6
		52.0	56.3	60.4	62.1
1.5		48.7	49.0	49.6	50.8
		41.3	45.2	49.2	50.6
2.0		21.6	21.9	22.3	23.2
		17.0	19.3	21.8	22.8
3.0		8.64	8.80	9.09	9.59
		6.7	7.71	8.92	9.2
5.0		3.73	3.82	3.98	4.25
		2.96	3.34	3.86	4.22

TABLE 4 The part data of X_1, X_2 and T_{CSDW} after being standardized

No.		**Data Sample**					T_{CSDW}
1	X_1	-0.32	0.14	-1.99	1.86	-0.84	
	X_2	-1.2	-0.04	-0.02	0.47	-1.08	11.9
2		-0.42	-0.62	1.14	-1.43	0.38	
		-0.96	0.32	0.36	-0.57	-1.83	5.25
…		…	…	…	…	…	…
11		-0.35	1.01	0.87	-1.27	-0.29	
		0.52	2.01	1.57	-0.69	1.37	40.07
…		…	…	…	…	…	…
20		1.95	2.5	-0.29	-2.28	1.73	
		1.51	1.82	0.35	-1.22	2.26	54.98

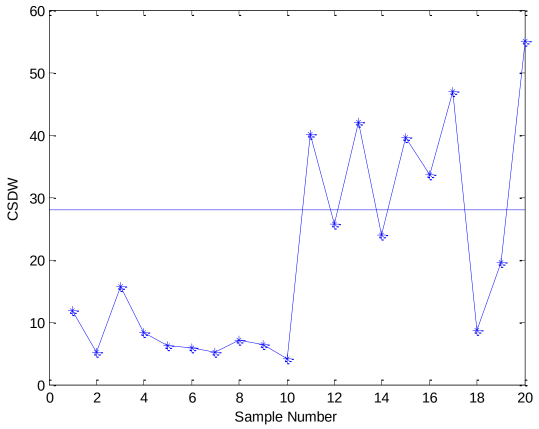

FIGURE 1 The CSDW chart for example

The part data of X_1, X_2 and T_{CSDW} are contained in Table 4 after being standardized. The UCL=28.05 (ARL_0=200) is determined by simulation. A probability of Type Ⅰ error of 5 per 1,000 $(\alpha = 0.005)$ is adopted.

We construct our CSDW chart in Figure 1. It is shown from Figure 1 that the process is out-of-control at the 11th sample point (Run length=1) because the point is above the UCL. We can investigate the unstable process and take actions to bring it back into an in-control state as quickly as possible.

6 Summary and Conclusions

In this study, the new bivariate single chart is capable of Monitoring the Mean Vector and Covariance Matrix Simultaneously. Overall, through comparisons the new chart performs better than the joint T^2 and $|\mathbf{S}|$ charts, the Max-MEWMA chart and the VMAX statistic. Meanwhile, the chart is easier to construct than the others because the chart does not require the need to select fewer parameters and requires less computation in designing. Practitioners could understand the procedure of the chart.

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